PROBLEMAS RESUELTOS DE CÁLCULO DE DERIVADAS

1) Calcular las derivadas de:

a) $f(x) = -\dfrac{2x^5}{\cos x}$

$$f'(x) = -\frac{10x^4 \cos x + 2x^5 \operatorname{sen} x}{\cos^2 x}$$

b) $g(x) = -\dfrac{3}{2}\ln\sqrt{7x}$

Simplificamos antes de derivar, aplicando propiedades de logaritmos neperianos: $g(x) = -\dfrac{3}{2}\dfrac{1}{2}\ln 7x = -\dfrac{3}{4}\ln 7x \Rightarrow g'(x) = -\dfrac{3}{4}\dfrac{7}{7x} = -\dfrac{3}{4x}$

c) $h(x) = \dfrac{e^{3x-5}}{2}$

Como $h(x) = \dfrac{1}{2}e^{3x-5} \Rightarrow h'(x) = \dfrac{1}{2}3e^{3x-5} = \dfrac{3}{2}e^{3x-5}$

2) Halle f '(2), g '(4) y h '(0) para las funciones definidas de la siguiente forma (L designa *logaritmo neperiano*):

$$f(x) = x^2 + \frac{16}{x^2}; \quad g(x) = (x^2+9)^3; \quad h(x) = \mathrm{L}(x^2+1).$$

Simplemente, aplicando las reglas de derivación, se obtiene:

$f'(x) = 2x - \dfrac{32x}{x^4} = 2x - \dfrac{32}{x^3}$ $\Rightarrow$ $f'(2) = 4 - \dfrac{32}{8} = 4-4 = 0$

$g'(x) = 3(x^2+9)^2 2x = 6x(x^2+9)^2$ $\Rightarrow$ $g'(4) = 24(16+9)^2 = 15.000$

$h'(x) = \dfrac{2x}{x^2+1}$ $\Rightarrow$ $h'(0) = \dfrac{0}{1} = 0$

3) Derivar y simplificar:

a) $f(x) = \dfrac{3x-1}{x} - (5x-x^2)^2$.

$$f'(x) = \frac{3x-(3x-1)}{x^2} - 2(5x-x^2)(5-2x) = \frac{1}{x^2} - 2(25x - 10x^2 - 5x^2 + 2x^3) =$$

$$= \frac{1}{x^2} - 2(2x^3 - 15x^2 + 25x) = \frac{1}{x^2} - 4x^3 + 30x^2 - 50x =$$

$$= \boxed{\frac{-4x^5 + 30x^4 - 50x^3 + 1}{x^2}}$$

b) $g(x) = (x^2-1)\cdot\ln x$.

$$g'(x) = 2x\ln x + \frac{x^2-1}{x} = \boxed{\frac{2x^2\ln x + x^2 - 1}{x}}$$

c) $h(x) = 2^{5x}$.

$$h'(x) = \boxed{5 \cdot 2^{5x} \ln 2}$$

d) $i(x) = (x^3 - 6x) \cdot (x^2 + 1)^3$.

$$i'(x) = (3x^2 - 6)(x^2 + 1)^3 + (x^3 - 6x)\ 3 \cdot 2x\ (x^2 + 1)^2 =$$
$$= (x^2 + 1)^2\ ((3x^2 - 6)\ (x^2 + 1) + 6x(x^3 - 6x)) =$$
$$= (x^2 + 1)^2\ (3x^4 + 3x^2 - 6x^2 - 6 + 6x^4 - 36x^2) = \boxed{(x^2 + 1)^2\ (9x^4 - 39x^2 - 6)}$$

e) $j(x) = (x+1) \cdot e^{2x+1}$.

$$j'(x) = e^{2x+1} + (x + 1)\ 2\ e^{2x+1} = e^{2x+1}(1 + 2x + 2) = \boxed{e^{2x+1}(2x + 3)}$$

f) $k(x) = 3x \cos 3x^2$.

$$k'(x) = 3 \cos 3x^2 - 3x \cdot 6x \operatorname{sen} 3x^2 = \boxed{3 \cos 3x^2 - 18x^2 \operatorname{sen} 3x^2}$$

Nota: La expresión simplificada final siempre puede resultar subjetiva, y debe entenderse como una expresión cómoda para operar y para volver a derivar si es preciso. Por ejemplo, en el d y el f se podría extraer 3 factor común.

4) Calcule las derivadas de las siguientes funciones:

a) $f(x) = \left(\frac{2-5x}{3}\right)^2 + \frac{1-2x}{x^2}$

$$f'(x) = 2\left(\frac{2-5x}{3}\right)\frac{-5}{3} + \frac{-2x^2 - (1-2x)2x}{x^4} = \frac{-10(2-5x)}{9} + \frac{-2x^2 - 2x + 4x^2}{x^4} =$$
$$= \frac{50x-20}{9} + \frac{2x^2 - 2x}{x^4} = \frac{50x-20}{9} + \frac{x(2x-2)}{x^4} = \frac{50x-20}{9} + \frac{2x-2}{x^3} =$$
$$= \frac{50x^4 - 20x^3}{9x^3} + \frac{18x-18}{9x^3} = \boxed{\frac{50x^2 - 20x^3 + 18x - 18}{9x^3}}$$

b) $g(x) = (3x + 2)^2 \ln(1 + x^2)$

$$g'(x) = 2(3x + 2)3\ln(1 + x^2) + (3x + 2)^2 \frac{2x}{1+x^2} =$$
$$= \boxed{(18x + 12)\ln(1 + x^2) + \frac{2x(3x+2)^2}{1+x^2}}$$

c) $h(x) = 2^{5x} + \frac{1}{x^2}$

$$h'(x) = \boxed{5 \cdot 2^{5x} \ln 2 - \frac{2}{x^3}}$$

5) Calcule las derivadas de las siguientes funciones:

a) $f(x) = \frac{3x-1}{x} - (5x - x^2)^2$.

$$f'(x) = \frac{3x - (3x-1)}{x^2} - 2(5x - x^2)(5 - 2x) = \frac{3x - 3x + 1}{x^2} - (5x - x^2)(10 - 4x) =$$

$$= \frac{1}{x^2} - (50x - 20x^2 - 10x^2 + 4x^3) = \frac{1 - 50x^2 + 30x^4 - 4x^5}{x^2} =$$

$$= \boxed{\frac{-4x^5 + 30x^4 - 50x^2 + 1}{x^2}}$$

b) $g(x) = (x^2 - 1) \cdot \ln x$.

$$g'(x) = \boxed{2x \ln x + \frac{x^2 - 1}{x}}$$

c) $h(x) = 2^{3x}$.

$$h'(x) = \boxed{3 \cdot 2^{3x} \ln 2}$$

d) $i(x) = (x^3 - 6x) \cdot (x^2 + 1)^3$.

$$i'(x) = (3x^2 - 6)(x^2 + 1)^3 + (x^3 - 6x)3(x^2 + 1)^2 2x =$$
$$= (3x^2 - 6)(x^2 + 1)^3 + (6x^4 - 36x^2)(x^2 + 1)^2 =$$
$$= (x^2 + 1)^2[(3x^2 - 6)(x^2 + 1) + 6x^4 - 36x^2] =$$
$$= (x^2 + 1)^2(3x^4 + 3x^2 - 6x^2 - 6 + 6x^4 - 36x^2) =$$
$$= \boxed{(x^2 + 1)^2(9x^4 - 39x^2 - 6)}$$

6) Calcular las derivadas de:

a) $y = \frac{\text{sen } x}{1 + \cos x} \Rightarrow y' = \frac{\cos x(1 + \cos x) - \text{sen } x(-\text{sen } x)}{(1 + \cos x)^2} =$

$$= \frac{\cos x + \cos^2 x + \text{sen}^2 x}{(1 + \cos x)^2} = \frac{\cos x + 1}{(1 + \cos x)^2} = \frac{(1 + \cos x)}{(1 + \cos x)^2} = \boxed{\frac{1}{1 + \cos x}}$$

b) $y = \text{arctg}(e^{-2x}) \Rightarrow y' = \frac{-2e^{-2x}}{1 + (e^{-2x})^2} = \boxed{-\frac{2e^{-2x}}{1 + e^{-4x}}}$

c) $y = \text{sen}^3 3x \Rightarrow y' = 3\,(\text{sen}^2 3x)\,(3 \cos 3x) = \boxed{9 \text{ sen}^2 3x \cos 3x}$

d) $y = \ln \frac{(x-2)^3}{\sqrt{2x-1}} = \ln (x-2)^3 - \ln \sqrt{2x-1} = 3 \ln(x-2) - \frac{1}{2} \ln(2x-1) \Rightarrow$

$$y' = 3\frac{1}{x-2} - \frac{1}{2}\frac{2}{2x-1} = \frac{3}{x-2} - \frac{1}{1}\frac{1}{2x-1} = \boxed{\frac{3}{x-2} - \frac{1}{2x-1}}$$

e) $y = x^3 e^{-3x} \Rightarrow y' = 3x^2 e^{-3x} + x^3 (-3) e^{-3x} = 3x^2 e^{-3x} - 3x^3 e^{-3x} =$

$$= \boxed{3x^2 e^{-3x} (1 - x)}$$

7) Derivar y simplificar: $y = \text{arctg } 3x^2$; $y = \frac{x^2 e^{1-x}}{3}$; $y = \ln \sqrt[3]{\frac{(x-2)^2}{x-3}}$; $y = 2\cos^2 4x$

- $y = \text{arctg } 3x^2 \Rightarrow y' = \frac{6x}{1 + (3x^2)^2} = \boxed{\frac{6x}{1 + 9x^4}}$
- $y = \frac{x^2 e^{1-x}}{3} = \frac{1}{3} x^2 e^{1-x} \Rightarrow y' = \frac{1}{3}(2xe^{1-x} + x^2(-1)e^{1-x}) = \boxed{\frac{xe^{1-x}(2-x)}{3}}$
- $y = \ln \sqrt[3]{\frac{(x-2)^2}{x-3}} = \frac{1}{3} \ln \frac{(x-2)^2}{x-3} = \frac{1}{3}\left[\ln(x-2)^2 - \ln(x-3)\right] =$

$$= \frac{1}{3}\left[2\ln(x-2) - \ln(x-3)\right] \Rightarrow y' = \frac{1}{3}\left(2\frac{1}{x-2} - \frac{1}{x-3}\right) =$$

$$= \boxed{\frac{2}{3(x-2)} - \frac{1}{3(x-3)}}$$

- $y = 2\cos^2 4x \Rightarrow y' = 2\cdot 2 (\cos 4x)(-\text{sen } 4x) 4 = \boxed{-16 \text{ sen } 4x \cos 4x} =$
  $= -8 \cdot 2 \text{ sen } 4x \cos 4x = -8 \text{ sen } (2\cdot 4x) = \boxed{-8 \text{ sen } 8x}$

8) Derivar y simplificar:

a) $y = e^{2x} \text{ tg } x \Rightarrow y' = 2e^{2x} \text{ tg } x + e^{2x}(1 + \text{tg}^2 x) = \boxed{e^{2x}(\text{tg}^2 x + 2 \text{ tg } x + 1)}$

b) $y = \ln \sqrt[3]{\frac{x^2}{x^2-4}} = \frac{1}{3}\left(2\ln x - \ln(x^2-4)\right) \Rightarrow y' = \boxed{\frac{1}{3}\left(\frac{2}{x} - \frac{2x}{x^2-4}\right)}$

c) $y = 2\cos^3 3x \Rightarrow y' = 2\cdot 3 \cos^2 3x (-\text{ sen } 3x) \cdot 3 = \boxed{-18 \text{ sen } 3x \cos^2 3x}$

d) $y = \text{arcsen } x^3 \Rightarrow y' = \frac{3x^2}{\sqrt{1-(x^3)^2}} = \boxed{\frac{3x^2}{\sqrt{1-x^6}}}$

9) Derivar las siguientes funciones, simplificando los resultados:

a) $y = 2e^{\cos 3x} \Rightarrow y' = 2e^{\cos 3x}(-\text{sen } 3x)3 = \boxed{-6\, e^{\cos 3x} \text{ sen } 3x}$

b) $y = \text{arctg } \sqrt{2x} \Rightarrow$

$$y' = \frac{\frac{2}{2\sqrt{2x}}}{1+\left(\sqrt{2x}\right)^2} = \frac{\frac{1}{\sqrt{2x}}}{1+2x} = \frac{\frac{1}{\sqrt{2x}}\frac{\sqrt{2x}}{\sqrt{2x}}}{1+2x} = \frac{\frac{\sqrt{2x}}{2x}}{1+2x} = \frac{\sqrt{2x}}{2x(1+2x)} = \boxed{\frac{\sqrt{2x}}{4x^2+2x}}$$

c) $y = \ln \sqrt[3]{\frac{(2x-3)^2}{x-3}} = \frac{1}{3}\ln\frac{(2x-3)^2}{x-3} = \frac{1}{3}\left[\ln(2x-3)^2 - \ln(x-3)\right] =$

$= \frac{1}{3}\left[2\ln(2x-3) - \ln(x-3)\right]$. Derivando:

$$y' = \frac{1}{3}\left[2\frac{2}{2x-3} - \frac{1}{x-3}\right] = \boxed{\frac{1}{3}\left[\frac{4}{2x-3} - \frac{1}{x-3}\right]}$$

d) $y = 3x \text{ tg } 4x \Rightarrow y' = 3 \text{ tg } 4x + 3x\frac{4}{\cos^2 4x} = \boxed{3 \text{ tg } 4x + \frac{12x}{\cos^2 4x}}$

e) $y = 2 \text{ sen}^2 3x \Rightarrow$

$y' = 4(\text{sen } 3x \cos 3x)\, 3 = 6 \cdot 2 \text{ sen } 3x \cos 3x = 6 \text{ sen } 2\cdot 3x = \boxed{6 \text{ sen } 6x}$

10) Derivar las siguientes funciones, simplificando los resultados:

a) $y = 2xe^{\text{sen } 3x}$

$y' = 2e^{\text{sen } 3x} + 2xe^{\text{sen } 3x}(\cos 3x)3 = \boxed{2e^{\text{sen } 3x}(1 + 3x\cos 3x)}$

b) $y = \ln \sqrt[3]{\frac{(4x-3)^2}{x-1}}$

Simplificando la expresión antes de derivar, aplicando propiedades de logaritmos:

$$y = \frac{1}{3}[\ln(4x-3)^2 - \ln(x-1)] = \frac{1}{3}[2\ln(4x-3) - \ln(x-1)] \Rightarrow$$

$$\Rightarrow y' = \frac{1}{3}\left(2\frac{4}{4x-3} - \frac{1}{x-1}\right) = \boxed{\frac{1}{3}\left(\frac{8}{4x-3} - \frac{1}{x-1}\right)}$$

c) $y = 3x^{\cos 2x}$

Tomamos ln antes de derivar:

$$\ln y = \ln 3x^{\cos 2x} = \ln 3 + \ln x^{\cos 2x} = \ln 3 + \cos 2x \ln x$$

Derivando miembro a miembro:

$$\frac{y'}{y} = 0 + -2\text{sen } 2x \ln x + (\cos 2x)\frac{1}{x} = -2\text{sen } 2x \ln x + \frac{\cos 2x}{x} =$$

$$= \frac{-2x\text{sen } 2x \ln x + \cos 2x}{x}$$

$$\Rightarrow y' = 3x^{\cos 2x}\frac{-2x\text{sen } 2x \ln x + \cos 2x}{x} = \boxed{3x^{\cos 2x - 1}(\cos 2x - 2x\text{sen } 2x \ln x)}$$

d) $y = 2\ \text{sen}^2 3x$

$$y' = 2\cdot 2\ \text{sen } 3x\ (\cos 3x)\ 3 = 6 \cdot 2\ \text{sen } 3x \cos 3x = 6\ \text{sen } 2\cdot 3x = \boxed{6\ \text{sen } 6x}$$

11) Derivar las siguientes funciones, simplificando los resultados:

a) $y = 3xe^{\cos x^2}$

$$y' = 3e^{\cos x^2} + 3xe^{\cos x^2}2x(-\text{sen } x^2) = \boxed{3e^{\cos x^2}(1 - 2x^2 \text{ sen } x^2)}$$

b) $y = \ln \sqrt[5]{\frac{4x^2-3}{(x-1)^2}}$

Simplificando la expresión antes de derivar, aplicando propiedades de logaritmos:

$$y = \frac{1}{5}[\ln(4x^2-3) - \ln(x-1)^2] = \frac{1}{5}[\ln(4x^2-3) - 2\ln(x-1)] \Rightarrow$$

$$\Rightarrow y' = \frac{1}{5}\left(\frac{8x}{4x^2-3} - 2\frac{1}{x-1}\right) = \boxed{\frac{1}{5}\left(\frac{8x}{4x^2-3} - \frac{2}{x-1}\right)}$$

c) $y = \text{arctg } \sqrt{3x}$

$$y' = \frac{\frac{3}{2\sqrt{3x}}}{1+\left(\sqrt{3x}\right)^2} = \frac{\frac{3}{2\sqrt{3x}}}{1+3x} = \boxed{\frac{3}{2\sqrt{3x}(1+3x)}}$$

d) $y = 3\cos^2 5x$

$$y' = 3\cdot 2 \cos 5x\ (-5 \text{ sen } 5x) = -15\cdot 2 \text{ sen } 5x \cos 5x =$$

$$= -15 \text{ sen } 2\cdot 5x = \boxed{-15 \text{ sen } 10x}$$

12) Calcule las derivadas de las siguientes funciones:

$$f(x) = \frac{2^x + x^2}{x}; \quad g(x) = (x^2+1)^2 \cdot \ln(e^{3x}+4)$$

$$f'(x) = \frac{(2^x \ln 2 + 2x)x - (2^x + x^2)\cdot 1}{x^2} = \frac{2^x x \ln 2 + 2x^2 - 2^x - x^2}{x^2} = \boxed{\frac{2^x x \ln 2 + x^2 - 2^x}{x^2}}$$

$$g'(x) = 2(x^2+1)2x \ln(e^{3x}+4) + (x^2+1)^2 \frac{3e^{3x}}{e^{3x}+4} =$$

$$= \boxed{(4x^3+4x)\ln(e^{3x}+4) + \frac{3e^{3x}(x^2+1)^2}{e^{3x}+4}}$$

13) Derivar y simplificar:

a) $y = 2(7x^3 - 3x)^6$

$$y' = 2\cdot 6(7x^3-3x)^5(21x^2-3) = 12(21x^2-3)(7x^3-3x)^5 =$$

$$= \boxed{(252x^2-36)(7x^3-3x)^5}$$

b) $y = \dfrac{3x^2-12}{x-1}$

$$y' = \frac{6x(x-1)-(3x^2-12)\cdot 1}{(x-1)^2} = \frac{6x^2-6x-3x^2+12}{(x-1)^2} = \boxed{\frac{3x^2-6x+12}{(x-1)^2}}$$

c) $y = \sqrt{2x^2+1}$

$$y' = \frac{4x}{2\sqrt{2x^2+1}} = \boxed{\frac{2x}{\sqrt{2x^2+1}}}$$

d) $y = (x+1)e^{2x+1}$

$$y' = 1\cdot e^{2x+1} + (x+1)2e^{2x+1} = e^{2x+1}[1+2(x+1)] = e^{2x+1}(1+2x+2) =$$

$$= \boxed{e^{2x+1}(2x+3)}$$

14) Derivar y simplificar:

a) $y = 2(7x^2 - 3x)^5$

$$y' = 2\cdot 5(7x^2-3x)^4(14x-3) = \boxed{10(14x-3)(7x^2-3x)^4}$$

b) $y = \dfrac{x-1}{3x^4-2}$

$$y' = \frac{3x^4-2-12x^3(x-1)}{(3x^4-2)^2} = \frac{3x^4-2-12x^4+12x^3}{(3x^4-2)^2} = \boxed{\frac{-9x^4+12x^3-2}{(3x^4-2)^2}}$$

c) $y = \text{sen}\,\sqrt{2x}$

$$y' = \frac{2}{2\sqrt{2x}}\cos\sqrt{2x} = \boxed{\frac{\cos\sqrt{2x}}{\sqrt{2x}}}$$

d) $y = e^{2x+1}\ln 3x$

$$y' = 2e^{2x+1}\ln 3x + e^{2x+1}\frac{3}{3x} = e^{2x+1}\left(2\ln 3x + \frac{1}{x}\right) = \boxed{e^{2x+1}\frac{1+2x\ln 3x}{x}}$$

15) Derivar: $y=\ln\dfrac{(x-2)^3}{\sqrt{2x-1}}$; $y= e^{2x}(3x^4+1)$ *(1 punto)*

- $y=\ln\dfrac{(x-2)^3}{\sqrt{2x-1}}$ $\Rightarrow$ La simplificamos antes de proceder a derivar:

  $y = \ln (x-2)^3 - \ln \sqrt{2x-1} = 3 \ln (x-2) - \dfrac{1}{2}\ln(2x-1)$. De donde:

  $$y' = 3\frac{1}{x-2}-\frac{1}{2}\frac{2}{2x-1} = \boxed{\frac{3}{x-2}-\frac{1}{2x-1}} = \frac{3(2x-1)-(x-2)}{(x-2)(2x-1)} = \frac{6x-3-x+2}{(x-2)(2x-1)} =$$

  $$= \boxed{\frac{5x-1}{(x-2)(2x-1)}} = \frac{5x-1}{2x^2-x-4x+2} = \boxed{\frac{5x-1}{2x^2-5x+2}}$$

  Cualquiera de las tres expresiones recuadradas valdría como final.

- $y= e^{2x}(3x^4+1) \Rightarrow y' = 2e^{2x}(3x^4+1)+e^{2x}12x^3 = e^{2x}[2(3x^4+1)+12x^3] =$

  $$= e^{2x}(6x^4+2+12x^3) = \boxed{e^{2x}(6x^4+12x^3+2)}$$

16) Derivar y simplificar: $y=\cos^3 2x$; $y=x\ln\sqrt{x-1}$; $y=\text{arcsen}\,(x-1)$; $y=e^{3\sqrt{x}}$

a) $y=\cos^3 2x \Rightarrow y' = 3\cos^2 2x\,(-2\,\text{sen}\,2x) = \boxed{-6\,\text{sen}\,2x\cos^2 2x}$

b) $y=x\ln\sqrt{x-1} = \dfrac{1}{2}x\ln(x-1) \Rightarrow y' = \dfrac{1}{2}\ln(x-1)+\dfrac{1}{2}x\dfrac{1}{x-1} =$

$$= \boxed{\frac{1}{2}\left(\ln(x-1)+\frac{x}{x-1}\right)}$$

c) $y=\text{arcsen}\,(x-1) \Rightarrow y' = \dfrac{1}{\sqrt{1-(x-1)^2}} = \dfrac{1}{\sqrt{1-(x^2-2x+1)}} = \dfrac{1}{\sqrt{1-x^2+2x-1}} =$

$$= \boxed{\frac{1}{\sqrt{2x-x^2}}}$$

d) $y=e^{3\sqrt{x}} \Rightarrow y' = 3\dfrac{1}{2\sqrt{x}}e^{3\sqrt{x}} = \boxed{\dfrac{3e^{3\sqrt{x}}}{2\sqrt{x}}}$

17) Derivar y simplificar:

a) $f(x) = \dfrac{3x+1}{(x-2)^2} \Rightarrow f'(x) = \dfrac{3(x-2)^2-(3x+1)2(x-2)}{(x-2)^4} =$

$$= \frac{(x-2)[3(x-2)-(3x+1)2]}{(x-2)^4} = \frac{3x-6-6x-2}{(x-2)^3} = \boxed{\frac{-3x-8}{(x-2)^3}}$$

b) $g(x) = \dfrac{3x}{\sqrt[3]{3x}}$

$$g(x) = \frac{3x}{\sqrt[3]{3x}} = \frac{3x}{\sqrt[3]{3x}}\frac{\sqrt[3]{3^2x^2}}{\sqrt[3]{3^2x^2}} = \frac{3x\sqrt[3]{3^2x^2}}{\sqrt[3]{3^3x^3}} = \frac{3x\sqrt[3]{3^2x^2}}{3x} = \sqrt[3]{3^2x^2} = \sqrt[3]{9x^2} \Rightarrow$$

$$g'(x) = \frac{18x}{3\sqrt[3]{(9x^2)^2}} = \frac{6x}{\sqrt[3]{3^4x^4}} = \frac{6x}{3x\sqrt[3]{3x}} = \boxed{\frac{2}{\sqrt[3]{3x}}}$$

De otra forma: $g(x) = \dfrac{3x}{\sqrt[3]{3x}} = (3x)^{1-\frac{1}{3}} = (3x)^{\frac{2}{3}} \Rightarrow g'(x) = \dfrac{2}{3}(3x)^{\frac{2}{3}-1}\cdot 3 = 2(3x)^{-\frac{1}{3}} =$

$$= \frac{2}{(3x)^{1/3}} = \boxed{\frac{2}{\sqrt[3]{3x}}}$$

c) $h(x) = 2xe^{5x^2} \Rightarrow h'(x) = 2\,e^{5x^2} + 2x\,10x\,e^{5x^2} = \boxed{2\,e^{5x^2}\,(1+10x^2)}$

d) $j(x) = \ln\sqrt[5]{\dfrac{(4x-1)^3}{3x^2-1}}$

$$j(x) = \ln\sqrt[5]{\frac{(4x-1)^3}{3x^2-1}} = \frac{1}{5}\ln\frac{(4x-1)^3}{3x^2-1} = \frac{1}{5}[\ln(4x-1)^3 - \ln(3x^2-1)] =$$

$$= \frac{1}{5}[3\ln(4x-1) - \ln(3x^2-1)] \Rightarrow j'(x) = \boxed{\frac{1}{5}\left(\frac{12}{4x-1} - \frac{6x}{3x^2-1}\right)}$$

18) Calcular y simplificar las derivadas de las siguientes funciones:

a) $f(x) = \dfrac{2(3x+1)^2}{3x-1}$

$$f'(x) = \frac{4(3x+1)3(3x-1) - 2(3x+1)^2 3}{(3x-1)^2} = \frac{(3x+1)[12(3x-1) - 6(3x+1)]}{(3x-1)^2} =$$

$$= \frac{(3x+1)(36x-12-18x-6)}{(3x-1)^2} = \boxed{\frac{(3x+1)(18x-18)}{(3x-1)^2}} = \frac{18(3x+1)(x-1)}{(3x-1)^2} =$$

$$= \frac{18(3x^2-3x+x-1)}{(3x-1)^2} = \boxed{\frac{18(3x^2-2x-1)}{(3x-1)^2}}$$

b) $g(x) = (x^2 - x + 1)\,e^{5x}$

$$g'(x) = (2x-1)\,e^{5x} + 5(x^2 - x + 1)\,e^{5x} = e^{5x}(2x - 1 + 5x^2 - 5x + 5) =$$

$$= \boxed{e^{5x}(5x^2 - 3x + 4)}$$

c) $j(x) = \ln\left(\sqrt[5]{\dfrac{(5x-3)^3}{2x^4}}\right)$

Simplificamos antes de derivar, aplicando propiedades de logaritmos:

$$j(x) = \ln\left(\sqrt[5]{\frac{(5x-3)^3}{2x^4}}\right) = \frac{1}{5}\ln\left(\frac{(5x-3)^3}{2x^4}\right) = \frac{1}{5}[\ln(5x-3)^3 - \ln(2x^4)] =$$

$$= \frac{1}{5}[3\ln(5x-3) - (\ln(2) + \ln(x^4))] = \frac{1}{5}[3\ln(5x-3) - \ln(2) - 4\ln(x)]$$

Y derivamos:

$$j'(x) = \frac{1}{5}\left[3\frac{5}{5x-3} - 0 - 4\frac{1}{x}\right] = \frac{1}{5}\frac{3\cdot 5}{5x-3} - \frac{1}{5}\frac{4}{x} = \boxed{\frac{3}{5x-3} - \frac{4}{5x}}$$

d) $h(x) = 3\sqrt[3]{5x^2-1}$

$$h'(x) = 3\frac{10x}{3\sqrt[3]{(5x^2-1)^2}} = \boxed{\frac{10x}{\sqrt[3]{(5x^2-1)^2}}} = \frac{10x}{\sqrt[3]{(5x^2-1)^2}}\frac{\sqrt[3]{5x^2-1}}{\sqrt[3]{5x^2-1}} = \frac{10x\sqrt[3]{5x^2-1}}{\sqrt[3]{(5x^2-1)^3}} =$$

$$= \boxed{\frac{10x\sqrt[3]{5x^2-1}}{5x^2-1}}$$

19) Calcular y simplificar las derivadas de las siguientes funciones:

a) $y = e^{3x}\text{sen } x$

$$y' = 3e^{3x}\text{sen } x + e^{3x}\cos x = \boxed{e^{3x}(3\text{sen } x + \cos x)}$$

b) $y = \cos^2 4x$

$$y' = 2(\cos 4x)(-4 \text{ sen } 4x) = \boxed{-8 \text{ sen } 4x \cos 4x} = -4\cdot 2 \text{ sen } 4x \cos 4x =$$
$$= -4 \text{ sen } (2\cdot 4x) = \boxed{-4 \text{ sen } 8x}$$

c) $\text{arctg } 6x^2$

$$y' = \frac{12x}{1+(6x^2)^2} = \boxed{\frac{12x}{1+36x^4}}$$

d) $y = \ln\left(\sqrt[5]{\frac{(5x-3)^3}{2x^4}}\right)$

Simplificamos antes de derivar, aplicando propiedades de logaritmos:

$$j(x) = \ln\left(\sqrt[5]{\frac{(5x-3)^3}{2x^4}}\right) = \frac{1}{5}\ln\left(\frac{(5x-3)^3}{2x^4}\right) = \frac{1}{5}[\ln(5x-3)^3 - \ln(2x^4)] =$$
$$= \frac{1}{5}[3\ln(5x-3) - (\ln(2)+\ln(x^4))] = \frac{1}{5}[3\ln(5x-3) - \ln(2) - 4\ln(x)]$$

Y derivamos:

$$j'(x) = \frac{1}{5}\left[3\frac{5}{5x-3} - 0 - 4\frac{1}{x}\right] = \frac{1}{5}\frac{3\cdot 5}{5x-3} - \frac{1}{5}\frac{4}{x} = \boxed{\frac{3}{5x-3} - \frac{4}{5x}} = \boxed{\frac{12-5x}{5x(5x-3)}}$$

20) Calcular y simplificar las derivadas de las siguientes funciones: *(1,6 puntos)*

a) $y = 2^{3x}\cos x$

$$y' = 3\cdot 2^{3x}(\ln 2)\cos x - 2^{3x}\text{sen } x = \boxed{2^{3x}(3\cos x \ln 2 - \text{sen } x)}$$

b) $y = \cos 4x^2$

$$y' = \boxed{-8x \text{ sen } 4x^2}$$

c) $y = \sqrt[3]{5x-2}$

$$y' = \boxed{\frac{5}{3\sqrt[3]{(5x-2)^2}}} = \boxed{\frac{5\sqrt[3]{5x-2}}{3(5x-2)}}$$

d) $y = \log_2(3x^2+5)$

$$y' = \boxed{\frac{6x}{3x^2+5}\cdot\frac{1}{\ln 2}}$$

21) Derivar y simplificar las siguientes funciones:

a) $y = 2^x \cos(5x^3 - 2)$

$$\boxed{y' =} 2^x \cos(5x^3-2)\ln 2 - 2^x \text{ sen } (5x^3-2)\, 15x^2 =$$
$$= \boxed{2^x[\cos(5x^3-2)\ln 2 - 15x^2 \text{ sen } (5x^3-2)]}$$

b) $y = \ln \sqrt[5]{\dfrac{6x^3}{(5-2x)^4}}$

$$y = \ln \sqrt[5]{\frac{6x^3}{(5-2x)^4}} = \frac{1}{5}\ln\frac{6x^3}{(5-2x)^4} = \frac{1}{5}[\ln 6x^3 - \ln(5-2x)^4] =$$

$$= \frac{1}{5}[\ln 6 + \ln x^3 - 4\ln(5-2x)] = \frac{1}{5}[\ln 6 + 3\ln x - 4\ln(5-2x)] \Rightarrow$$

$$\boxed{y' =} \frac{1}{5}\left[0 + \frac{3}{x} - 4\frac{-2}{5-2x}\right] = \boxed{\frac{1}{5}\left[\frac{3}{x} + \frac{8}{5-2x}\right]}$$

c) $y = \text{arctg } e^{3x}$

$$\boxed{y' =} \frac{3e^{3x}}{1+(e^{3x})^2} = \boxed{\frac{3e^{3x}}{1+e^{6x}}}$$

d) $y = \log (2x^4 + 1)$

$$\boxed{y' = \frac{8x^3}{2x^4+1}\frac{1}{\ln 10}}$$

22) Derivar y simplificar las siguientes funciones:

a) $y = 2\sqrt[4]{4x^3}$

$$\boxed{y' =} 2\frac{12x^2}{4\sqrt[4]{(4x^3)^3}} = \frac{6x^2}{\sqrt[4]{4^3x^9}} = \frac{6x^2}{\sqrt[4]{(2^2)^3x^8x}} = \frac{6x^2}{x^2\sqrt[4]{2^6x}} = \frac{6}{2\sqrt[4]{2^2x}} = \boxed{\frac{3}{\sqrt[4]{4x}}}$$

b) $y = \text{tg } (5x^3 + 1) \Rightarrow \boxed{y' = \dfrac{15x^2}{\cos^2(5x^3+1)}}$

c) $y = e^x(4x^3 + 2)^3$

$$y' = e^x(4x^3 + 2)^3 + e^x\, 3(4x^3 + 2)^2\, 12x^2 = e^x\, (4x^3 + 2)^2\, [(4x^3 + 2) + 36x^2] =$$

$$= \boxed{e^x\, (4x^3 + 2)^2\, (4x^3 + 36x^2 + 2)}$$

d) $y = \ln \sqrt[5]{\dfrac{2x^3}{(5-2x)^3}} = \dfrac{1}{5}\ln\dfrac{2x^3}{(5-2x)^3} = \dfrac{1}{5}\left[\ln 2x^3 - \ln(5-2x)^3\right] =$

$$= \frac{1}{5}[\ln 2 + \ln x^3 - 3\ln(5-2x)] = \frac{1}{5}[\ln 2 + 3\ln x - 3\ln(5-2x)] \Rightarrow$$

$$y' = \frac{1}{5}\left(0 + \frac{3}{x} - 3\frac{-2}{5-2x}\right) = \boxed{\frac{1}{5}\left(\frac{3}{x} + \frac{6}{5-2x}\right)}$$

23) Calcule las derivadas de las siguientes funciones:

a) $f(x) = \dfrac{(x^2-5)^4}{3-x^2}$ b) $g(x) = e^{7x}(x - 5x^2)^3$ c) $h(x) = \dfrac{x\cdot\ln(x^2-1)}{x-2}$

- $f(x) = \dfrac{(x^2-5)^4}{3-x^2} \Rightarrow$

$$f'(x) = \frac{4(x^2-5)^3 2x(3-x^2) - (x^2-5)^4(-2x)}{(3-x^2)^2} =$$

$$= \frac{8x(x^2-5)^3(3-x^2)+2x(x^2-5)^4}{(3-x^2)^2} = \frac{(x^2-5)^3[8x(3-x^2)+2x(x^2-5)]}{(3-x^2)^2} =$$

$$= \frac{(x^2-5)^3[24x-8x^3+2x^3-10x]}{(3-x^2)^2} = \boxed{\frac{(x^2-5)^3(-6x^3+14x)}{(3-x^2)^2}}$$

- $g(x) = e^{7x}(x - 5x^2)^3 \quad 3 \Rightarrow$

  $g'(x) = 7e^{7x}(x - 5x^2)^3 + e^{7x}\, 3(x - 5x^2)^2(1 - 10x) =$

  $= e^{7x}(x - 5x^2)^2\, [7(x - 5x^2) + 3(1 - 10x)] = e^{7x}(x - 5x^2)^2\, [7x - 35x^2 + 3 - 30x] =$

  $= \boxed{e^{7x}(x - 5x^2)^2\, (- 35x^2 - 23x + 3)}$

- $h(x) = \dfrac{x \cdot \ln(x^2-1)}{x-2} \Rightarrow$

  $$h'(x) = \frac{\left[\ln(x^2-1)+x\dfrac{2x}{x^2-1}\right](x-2)-x\ln(x^2-1)}{(x-2)^2} =$$

  $$= \frac{\dfrac{(x^2-1)\ln(x^2-1)+2x^2}{x^2-1}(x-2)-x\ln(x^2-1)}{(x-2)^2} =$$

  $$= \frac{\dfrac{(x^2-1)(x-2)\ln(x^2-1)+2x^2(x-2)-x(x^2-1)\ln(x^2-1)}{x^2-1}}{(x-2)^2} =$$

  $$= \boxed{\frac{(x^2-1)(x-2)\ln(x^2-1)+2x^2(x-2)-x(x^2-1)\ln(x^2-1)}{(x^2-1)(x-2)^2}}$$

24) Calcule las derivadas de las siguientes funciones:

a) $f(x) = e^{3x}\cdot\ln(2x - 5)$ b) $g(x) = \dfrac{3^{2x}}{x^2-1}$ c) $h(x) = (3x^2 + 5x - 1)^6 + x^2 - \ln x$

a) $f(x) = e^{3x}\cdot\ln(2x - 5)$

$$f'(x) = 3e^{3x}\ln(2x - 5) + e^{3x}\frac{2}{2x-5} = \boxed{e^{3x}\left(3\ln(2x-5)+\frac{2}{2x-5}\right)}$$

b) $g(x) = \dfrac{3^{2x}}{x^2-1}$

$$g'(x) = \boxed{\frac{2\cdot 3^{2x}(\ln 3)(x^2-1)-2x\cdot 3^{2x}}{(x^2-1)^2}}$$

c) $h(x) = (3x^2 + 5x - 1)^6 + x^2 - \ln x$

$$h'(x) = \boxed{6(3x^2 + 5x - 1)^5(6x + 5) + 2x - \frac{1}{x}}$$

25) Derivar: $g(x) = (x^2 + 1)^2 \cdot \ln(e^{3x} + 4)$

$$\boxed{g'(x)} = 2(x^2 + 1)2x\ln(e^{3x} + 4) + (x^2 + 1)^2\frac{3e^{3x}}{e^{3x}+4} =$$

$$= \boxed{(4x^3 + 4x)\ln(e^{3x} + 4) + \frac{3e^{3x}(x^2+1)^2}{e^{3x}+4}}$$

26) Derivar y simplificar:

a) $y = \dfrac{3x^2+1}{(2x^3-3)^2}$ b) $y = 2^{7x-3}\ln(x^4+1)$

a) $y' = \dfrac{6x(2x^3-3)^2 - (3x^2+1)2(2x^3-3)6x^2}{(2x^3-3)^4} =$

$$= \frac{(2x^3-3)[6x(2x^3-3)-(3x^2+1)12x^2]}{(2x^3-3)^4} = \frac{12x^4-18x-36x^4-12x^2}{(2x^3-3)^3} =$$

$$= \boxed{\frac{-24x^4-12x^2-18x}{(2x^3-3)^3}}$$

b) $y' = 7\cdot 2^{7x-3}\ln 2\ln(x^4+1) + 2^{7x-3}\dfrac{4x^3}{x^4+1} = \boxed{2^{7x-3}\left(7\ln 2\ln(x^4+1)+\dfrac{4x^3}{x^4+1}\right)}$

27) Derivar y simplificar:

a) $y = \dfrac{5x^2+1}{(2x^3-3)^2}$ b) $y = 3^{7x-3}\ln(x^4+1)$

a) $y' = \dfrac{10x(2x^3-3)^2 - (5x^2+1)2(2x^3-3)6x^2}{(2x^3-3)^4} =$

$$= \frac{(2x^3-3)[10x(2x^3-3)-(5x^2+1)12x^2]}{(2x^3-3)^4} = \frac{20x^4-30x-60x^4-12x^2}{(2x^3-3)^3} =$$

$$= \boxed{\frac{-40x^4-12x^2-30x}{(2x^3-3)^3}}$$

b) $y' = 7\cdot 3^{7x-3}\ln 3\ln(x^4+1) + 3^{7x-3}\dfrac{4x^3}{x^4+1} = \boxed{3^{7x-3}\left(7\ln 3\ln(x^4+1)+\dfrac{4x^3}{x^4+1}\right)}$

28) Derivar y simplificar:

a) $f(x) = \dfrac{2^{5x}}{(x^3-1)^2}$ b) $g(x) = 4x^3\ln(3x+1)$

a) $f(x) = \dfrac{2^{5x}}{(x^3-1)^2} \Rightarrow f'(x) = \dfrac{5\cdot 2^{5x}(\ln 2)(x^3-1)^2 - 2^{5x}2(x^3-1)3x^2}{(x^3-1)^4} =$

$$= \frac{(x^3-1)2^{5x}[5(\ln 2)(x^3-1)-6x^2]}{(x^3-1)^4} = \boxed{\frac{2^{5x}[(5x^3-5)\ln 2 - 6x^2]}{(x^3-1)^3}}$$

b) $g(x) = 4x^3\ln(3x+1) \Rightarrow g'(x) = 12x^2\ln(3x+1) + \dfrac{4x^3 3}{3x+1} =$

$$= \boxed{12x^2\ln(3x+1) + \frac{12x^3}{3x+1}}$$

29) Derivar y simplificar:

a) $f(x) = \ln \dfrac{x}{2x^3+1}$ b) $g(x) = \dfrac{2^{5x}}{(x^4-1)^2}$

a) Simplificamos antes de derivar: $f(x) = \ln \dfrac{x}{2x^3+1} = \ln x - \ln(2x^3+1)$. Ahora derivamos: $f'(x) = \dfrac{1}{x} - \dfrac{6x^2}{2x^3+1} = \dfrac{2x^3+1-6x^3}{2x^4+x} = \boxed{\dfrac{-4x^3+1}{2x^4+x}}$

b) $g(x) = \dfrac{2^{5x}}{(x^4-1)^2} \Rightarrow g'(x) = \dfrac{5\cdot 2^{5x}(\ln 2)(x^4-1)^2 - 2^{5x}2(x^4-1)4x^3}{(x^4-1)^4} =$

$$= \frac{2^{5x}(x^4-1)[5(\ln 2)(x^4-1) - 2\cdot 4x^3]}{(x^4-1)^4} = \boxed{\frac{2^{5x}[(5x^4-5)\ln 2 - 8x^3]}{(x^4-1)^3}}$$

30) Derivar y simplificar:

a) $f(x) = \dfrac{3^{2x+1}}{(2x-1)^2}$ b) $g(x) = (4x^3 - 6x)^2 \ln x$

a) $f'(x) = \dfrac{2\cdot 3^{2x+1}(\ln 3)(2x-1)^2 - 3^{2x+1}2(2x-1)2}{(2x-1)^4} = \dfrac{3^{2x+1}(2x-1)[2\cdot(\ln 3)(2x-1)-4]}{(2x-1)^4} =$

$$= \boxed{\frac{3^{2x+1}[(4x-2)\ln 3 - 4]}{(2x-1)^3}}$$

b) $g'(x) = 2(4x^3-6x)(12x^2-6)\ln x + (4x^3-6x)^2\dfrac{1}{x} =$

$$= (4x^3-6x)\left(2(12x^2-6)\ln x + \frac{4x^3-6x}{x}\right) = \boxed{(4x^3-6x)\left((24x^2-12)\ln x + 4x^2 - 6\right)}$$

31) Derivar y simplificar:

$$f(x) = \ln\left(\frac{x}{4x^3-2}\right) \qquad g(x) = \frac{3^{2x}}{2x^3-3}$$

Simplificamos la primera función antes de derivarla:

$f(x) = \ln\left(\dfrac{x}{4x^3-2}\right) = \ln x - \ln(4x^3-2) \Rightarrow f'(x) = \dfrac{1}{x} - \dfrac{12x^2}{4x^3-2} = \dfrac{4x^3-2-12x^3}{4x^4-2x} =$

$$= \frac{-8x^3-2}{4x^4-2x} = \frac{2(-4x^3-1)}{2(2x^4-x)} = \boxed{\frac{-4x^3-1}{2x^4-x}}$$

$g(x) = \dfrac{3^{2x}}{2x^3-3} \Rightarrow g'(x) = \dfrac{2\cdot 3^{2x}(\ln 3)(2x^3-3) - 3^{2x}6x^2}{(2x^3-3)^2} = \boxed{\dfrac{3^{2x}[(4x^3-6)\ln 3 - 6x^2]}{(2x^3-3)^2}}$

32) Derivar y simplificar:

a) $f(x) = \dfrac{1-x^2}{(x^3-1)^2}$

$$f'(x) = \frac{-2x(x^3-1)^2 - (1-x^2)2(x^3-1)3x^2}{(x^3-1)^4} = \frac{(x^3-1)[-2x(x^3-1)-(1-x^2)6x^2]}{(x^3-1)^4} =$$

$$= \frac{-2x^4 + 2x - 6x^2 + 6x^4}{(x^3-1)^3} = \boxed{\frac{4x^4 - 6x^2 + 2x}{(x^3-1)^3}}$$

b) $g(x) = (4x^2 - 3)e^{-3x^2}$

$$g'(x) = 8x\, e^{-3x^2} - 6x\, e^{-3x^2}(4x^2 - 3) = e^{-3x^2}(8x - 24x^3 + 18x) = \boxed{e^{-3x^2}(-24x^3 + 26x)}$$

c) $h(x) = 3\sqrt[3]{5x^2-1}$

$$h'(x) = 3\frac{10x}{3\sqrt[3]{(5x^2-1)^2}} = \boxed{\frac{10x}{\sqrt[3]{(5x^2-1)^2}}} = \frac{10x}{\sqrt[3]{(5x^2-1)^2}}\frac{\sqrt[3]{5x^2-1}}{\sqrt[3]{5x^2-1}} = \frac{10x\sqrt[3]{5x^2-1}}{\sqrt[3]{(5x^2-1)^3}} =$$

$$= \boxed{\frac{10x\sqrt[3]{5x^2-1}}{5x^2-1}}$$

d) $j(x) = \log(x^2 + x + 1)$

$$\boxed{j'(x) = \frac{1}{\ln 10}\frac{2x+1}{x^2+x+1}}$$

33) Derivar y simplificar:

a) $y = \dfrac{4-2x}{(3-x)^2}$

$$y' = \frac{-2(3-x)^2 - (4-2x)(-2(3-x))}{(3-x)^4} = \frac{(3-x)[-2(3-x) + (4-2x)2]}{(3-x)^4} =$$

$$= \frac{-6 + 2x + 8 - 4x}{(3-x)^3} = \boxed{\frac{2-2x}{(3-x)^3}}$$

b) $g(x) = (3 - 7x^2)e^{-3x^2}$

$$g'(x) = -14x\, e^{-3x^2} - 6x\, e^{-3x^2}(3 - 7x^2) = e^{-3x^2}(-14x - 18x + 42x^3) =$$

$$= \boxed{e^{-3x^2}(42x^3 - 32x)}$$

c) $h(x) = 2^{5x} + \dfrac{1}{x^2}$

$$\boxed{h'(x) = 5\cdot 2^{5x}\ln(2) - \frac{2}{x^3}}$$

d) $j(x) = \log(x^2 + 2x - 1)$

$$\boxed{j'(x) = \frac{1}{\ln 10}\frac{2x+2}{x^2+2x-1}}$$

34) Calcule y simplifique las derivadas de:

$f(x) = (x^2 - 1)\cdot(3x^3 + 5)^3$ $\qquad g(x) = \dfrac{\ln(2x)}{e^{3x}}$ $\qquad h(x) = \log(3x^2 - x)$

- $f(x) = (x^2 - 1)\cdot(3x^3 + 5)^3$

$$f'(x) = 2x\,(3x^3 + 5)^3 + (x^2 - 1)\,3\,(3x^3 + 5)^2\, 9x^2 =$$

$$= 2x\,(3x^3 + 5)^3 + 27x^2\,(x^2 - 1)\,(3x^3 + 5)^2 =$$

$$= (3x^3 + 5)^2\,[2x\,(3x^3 + 5) + 27x^2\,(x^2 - 1)] =$$

$$= (3x^3 + 5)^2\,(6x^4 + 10x + 27x^4 - 27x^2) = \boxed{(3x^3 + 5)^2\,(33x^4 - 27x^2 + 10x)}$$

- $g(x) = \dfrac{\ln(2x)}{e^{3x}}$

$$g'(x) = \frac{\frac{2}{2x}e^{3x} - \ln(2x)3e^{3x}}{(e^{3x})^2} = \frac{\frac{e^{3x}}{x} - \frac{x\ln(2x)3e^{3x}}{x}}{e^{6x}} = \frac{\frac{e^{3x} - 3xe^{3x}\ln(2x)}{x}}{e^{6x}} =$$

$$= \frac{e^{3x} - 3xe^{3x}\ln(2x)}{xe^{6x}} = \frac{e^{3x}(1 - 3x\ln(2x))}{xe^{6x}} = \frac{1 - 3x\ln(2x)}{xe^{6x-3x}} = \boxed{\frac{1 - 3x\ln(2x)}{xe^{3x}}}$$

- $h(x) = \log(3x^2 - x)$

$$h'(x) = \boxed{\frac{1}{\ln(10)}\frac{6x-1}{3x^2 - x}}$$

35) Calcular y simplificar las derivadas de las siguientes funciones:

a) $f(x) = \dfrac{1-3x}{x} + (5x-2)^3$.

$$f'(x) = \frac{-3x - (1-3x)}{x^2} + 3(5x-2)^2 5 = \frac{-3x - 1 + 3x}{x^2} + 15(5x-2)^2 =$$

$$= \boxed{\frac{-1}{x^2} + 15(5x-2)^2}$$

b) $g(x) = (x^2+2)\cdot\ln(x^2+2)$.

$$g'(x) = 2x\ln(x^2 + 2) + (x^2 + 2)\,\frac{2x}{x^2+2} = 2x\ln(x^2 + 2) + 2x = \boxed{2x[1 + \ln(x^2 + 2)]}$$

c) $h(x) = 3^{5x} + e^x$.

$$h'(x) = \boxed{5\cdot 3^{5x}\ln 3 + e^x}$$

36) Calcular y simplificar las derivadas de las siguientes funciones:

$$f(x) = \frac{2^x + x^2}{x}\,; \quad g(x) = (x^2 + 1)^2\cdot\ln(e^{3x} + 4)$$

$$f'(x) = \frac{(2^x\ln 2 + 2x)x - (2^x + x^2)\cdot 1}{x^2} = \frac{2^x x\ln 2 + 2x^2 - 2^x - x^2}{x^2} = \boxed{\frac{2^x x\ln 2 + x^2 - 2^x}{x^2}}$$

$$g'(x) = 2(x^2 + 1)2x\ln(e^{3x} + 4) + (x^2 + 1)^2\,\frac{3e^{3x}}{e^{3x}+4} =$$

$$= \boxed{(4x^3 + 4x)\ln(e^{3x} + 4) + \frac{3e^{3x}(x^2+1)^2}{e^{3x}+4}}$$

37) Calcular y simplificar las derivadas de las siguientes funciones:

a) $f(x) = \dfrac{-2x^2 + x}{(1-x)^2}$

$$f'(x) = \frac{(-4x+1)(1-x)^2-(-2x^2+x)2(1-x)(-1)}{(1-x)^4} =$$

$$= \frac{(1-x)[(-4x+1)(1-x)+2(-2x^2+x)]}{(1-x)^4} = \frac{-4x+4x^2+1-x-4x^2+2x}{(1-x)^3} = \boxed{\frac{-3x+1}{(1-x)^3}}$$

b) $g(x) = 2^{1-x^3}(1-x^3)^2$

$$g'(x) = 2^{1-x^3}(-3x^2)(\ln 2)(1-x^3)^2 + 2^{1-x^3}2(1-x^3)(-3x^2) =$$

$$= 2^{1-x^3}(-3x^2)(1-x^3)[(\ln 2)(1-x^3)+2] = \boxed{2^{1-x^3}(3x^5-3x^2)[(1-x^3)\ln 2+2]}$$

Hay otras posibilidades de simplificación, procedentes de sacar factor común, pero nos conformaremos con ésta.

c) $h(x) = \log(1-x^3)$

$$h'(x) = \boxed{\frac{-3x^2}{1-x^3}\,\frac{1}{\ln 10}}$$

38) Derivar y simplificar:

a) $f(x) = (x^3+1)\,e^{7x}$

$$\boxed{f'(x) =}\ 3x^2 e^{7x} + (x^3+1)\,7\,e^{7x} = \boxed{e^{7x}(7x^3+3x^2+7)}$$

b) $g(x) = 3^x \ln(2x+5)$

$$\boxed{g'(x) =}\ 3^x \ln 3 \ln(2x+5) + 3^x\frac{2}{2x+5} = \boxed{3^x\left(\ln 3\ln(2x+5)+\frac{2}{2x+5}\right)}$$

c) $h(x) = \dfrac{x^2-2}{(x+1)^2}$

$$\boxed{h'(x) =}\ \frac{2x(x+1)^2-(x^2-2)2(x+1)}{(x+1)^4} = \frac{(x+1)[2x(x+1)-2(x^2-2)]}{(x+1)^4} =$$

$$= \frac{2x^2+2x-2x^2+4}{(x+1)^3} = \boxed{\frac{2x+4}{(x+1)^3}}$$

39) Calcule las siguientes derivadas, simplificando los resultados:

a) $g'(3)$, siendo $g(x) = 2xe^{3x-1}$.

$$g'(x) = 2e^{3x-1} + 2x3e^{3x-1} = e^{3x-1}(2+6x) \Rightarrow \boxed{g'(3) = 20e^8}$$

b) $f(x) = \dfrac{1-x^2}{(x^3-1)^2}$

$$\boxed{f'(x) =}\ \frac{-2x(x^3-1)^2-(1-x^2)2(x^3-1)3x^2}{(x^3-1)^4} =$$

$$= \frac{(x^3-1)[-2x(x^3-1)-(1-x^2)6x^2]}{(x^3-1)^4} = \frac{-2x^4+2x-6x^2+6x^4}{(x^3-1)^3} = \boxed{\frac{4x^4-6x^2+2x}{(x^3-1)^3}}$$

c) $h(x) = \log(x^2 + x + 1)$

$$\boxed{h'(x) = \frac{2x+1}{x^2+x+1}\frac{1}{\ln 10}}$$

40) Calcular y simplificar las derivadas de las siguientes funciones:

a) $f(x) = \dfrac{1-3x}{x} + (5x-2)^3$.

$$f'(x) = \frac{-3x-(1-3x)}{x^2} + 3(5x-2)^2 5 = \frac{-3x-1+3x}{x^2} + 15(5x-2)^2 =$$

$$= \boxed{\frac{-1}{x^2} + 15(5x-2)^2}$$

b) $g(x) = \left(x^2+2\right)\cdot \ln\left(x^2+2\right)$.

$$g'(x) = 2x\ln(x^2+2) + (x^2+2)\frac{2x}{x^2+2} = 2x\ln(x^2+2) + 2x = \boxed{2x[1 + \ln(x^2+2)]}$$

c) $h(x) = 3^{5x} + e^x$.

$$h'(x) = \boxed{5\cdot 3^{5x}\ln 3 + e^x}$$